LA-UR -87-2961

CONF-870911--

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36

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LA-UR--87-2961

DE88 000510

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SUBMITTED TO Ninth International FEL Conference, Williamsburg, Virginia September 14-18, 1987

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SPECTRAL EVOLUTION EQUATION FOR THE SIDEBAND INSTABILITY IN A FREE-ELECTRON LASER OSCILLATOR*

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Assuming that the light remains in the optical system while the electrons pass through and are lost, we have derived the evolution equation for an arbitrary optical spectrum in a free-electron laser oscillator. The resulting integrodifferential equation includes the effects of linear optical system losses (due to either mirror reflectivity or clipping), the usual small signal gain term, and a quadratic nonlinear term that includes the effects of saturation and coupling to sideband modes. The effects of detuning and particle beam current and energy variations can easily be included. Results of numerical solutions of the equations for various conditions will be presented.

1. Introduction

The sideband instability is thought to be important in the dynamics of free-electron lasers (FELs) and in fact has been observed in the saturated state of the Los Alamos oscillator experiment [1]. Most analyses of the instability, including the original [2], have concentrated on long wigglers (long enough that many synchrotron oscillations occur) and have assumed a sharp optical spectrum. Although this is the situation we would like to achieve, present FEL oscillator experiments are usually relatively short (less than one or two synchrotron oscillations occur) and display a broad optical spectrum [3]. Our purpose here is to outline a theory that describes the present situation for FEL oscillators to better un

* Work performed under the auspices of the U.S. Dept. of Energy and supported by the U.S. Army Strategic Defense Command.

derstand experimental results with broad spectra and to develop for such experiments a tool to predict the effects of various modifications, such as irrors with enhanced damping at certain wavelength ranges. Although we are able in this work to allow more general optical spectra than in most treatments, we must pay the price of restricting the length of the wiggler to less than a couple of synchrotron oscillations long; i.e., we relax the restrictions on the light but increase the restric tions on the particles. Thus, our theory is aimed precisely at the present experiments but cannot be easily extended with confidence to future very high power devices.

In an FEL oscillator, the electron beam passes through the wiggler once and is either discarded or undergoes energy recovery while the optical beam is confined by the mirrors for a large number of passes. Thus, though an electron is not in the system long enough (in short wigglers) to undergo a full synchrotron oscillation, the light stays in the system and carries the information about the particle oscillations in such a way that a sideband instability can develop. This fact suggests that one should concentrate on the information carried by the optical beam rather than on the dynamics of the electrons in such FELs. This is the course we will take, though the interaction of the electrons in the wiggler with the optical beam is still included in a self-consistent manner. We will derive an evolution equation for the optical spectrum that includes the effects of the sideband instability and of nonlinear saturation. By considering the time-independent states of this equation, we can derive saturated states and will find good agreement between these results and the experiment.

Note that in this paper we will speak of the length of the wiggler in terms of the number of synchrotron oscillations an electron would underge in the presence of a single spectral component even though we are *not* assuming that the spectrum is narrow enough for synchrotron oscillations to be well defined. A narrow spectrum is, in some sense, the worst case for the validity of our theory, though it should be accurate for a small number of synchrotron oscillations even in such a case.

In the next section, we will present the physical model we use and will indicate how the derivation of the spectral equation proceeds in general. In the succeeding section, we will specialize the general theory to the case of a constant parameter wiggler, outline details of the particle trajectories description, present some general remarks on the dynamics that result, and compare the results for one case to the experiment. In the final section, we will discuss our results and indicate possible directions for future work.

2. General model

As in ref. [2], we will use a one-dimensional model of the FEL; we ignore the transverse variation of the light amplitude, the particle distribution function, and the wiggler magnetic field, though the occurrence of three-dimensional effects is now recognized as being important for long wigglers. We will use an oscillator model for the light; the light is assumed to remain in the system for a time long compared to the time for a single pass through the wiggler. Optical system losses such as mirror absorption and transmission and beam clipping are assumed to be described by a linear loss term, though this loss term is allowed to depend in an arbitrary manner on wavelength.

The particles are assumed to be very relativistic so that we can ignore nonresonant terms of order $1/\gamma^2$. In addition, we assume that the optical magnetic field B_s is not much larger than the wiggler magnetic field B_w (an assumption that remains true even at fairly high power) so that the associated vector potentials satisfy $|A_s| \ll |A_w|$. This assumption allows us to expand the particle motion in a series in A_s about $A_s = 0$. The zeroth order in this expansion is the free motion of electrons down the wiggler. The first order describes the lowest order interaction of the electrons with the light and allows us to derive a small signal gain formula that agrees with the conventional result. To include saturation and sideband effects, it is necessary to carry this expansion to third order, at which point the trajectories of the electrons including the interaction with the light are accurate enough to describe the motion for about one or two synchrotron oscillations.

The electron beam is given by a prescribed (possibly time-dependent) distribution at the beginning of each pass, which then evolves in accordance with the particle trajectories down the wiggler. Note that though the distribution is prescribed *a priori* at the beginning of each pass, usually in a time-independent manner, the evolution down the wiggler depends on the light spectrum at that pass and, thus, will usually be different from one pass to the next unless the spectrum is in a steady state. We will also assume that the electron beam is not prebunched on the optical scale length at the wiggler entrance; this assumption is violated in certain recent wiggler designs that include an optical bunching section. Finally, we will ignore particle discreteness effects so that spontaneous emission and related effects are not included in our model.

To derive the evolution equation for the spectrum, we begin with Maxwell's equation for the optical vector potential

$$\nabla^2 \vec{A}_s - \frac{1}{c^2}\frac{\partial^2 \vec{A}_s}{\partial t^2} = -\frac{4\pi \vec{J}}{c}, \tag{1}$$

where, assuming the wiggler field is in the y-direction and the beam axis is in the z-direction, we expand the vector potential in modes of the optical system as

$$\vec{A}_s = \hat{x}\sum_n A_n(t)\cos[k_n(z - ct)] , \tag{2}$$

where the wave number $k_n = 2\pi n/L_s$, with L_s being the optical system length. Using this expansion in Maxwell's equation and assuming moderate gains so that second time derivatives of $A_n(t)$ can be ignored, we find that $A_n(t)$ satisfies

$$\sum_n k_n \dot{A}_n(t)\sin[k_n(z - ct)] = 2\pi J_x(z,t) , \tag{3}$$

where the dot indicates a time derivative. Multiplying by $\sin[k_m(z - ct)]$, integrating in t over a pass through the wiggler, and then integrating in z over the wiggler length L_W, we find that the change in a component of the vector potential in a pass ΔA_m is given by

$$\Delta A_m = -l_m A_m + \frac{4\pi}{k_m L_W}\int_0^{L_W} dz \int_0^T dt\, J_x(z,t)\sin[k_m(z-ct)], \tag{4}$$

where T is the time to traverse the wiggler ($T = L_W/c$) and l_m is the loss function for the optical system. Dividing by the time for the light to make a round trip in the system ($= 2L_s/c$), we find that

$$\frac{dA_m}{dt} \approx -\frac{c l_m}{2L_s}A_m + \frac{2\pi c}{k_m L_s L_W}\int_0^{L_W} dz \int_0^T dt\, J_x(z,t)\sin[k_m(z-ct)]\ . \tag{5}$$

The electron beam current $J_x(z,t)$ is given by

$$J_x(z,t) = -e\int dz_i d\gamma_0 f_0(z_i,\gamma_0) v_x(t,\gamma_0,z_i)\delta[z(t,\gamma_0,z_i) - z]\ , \tag{6}$$

where f_0 is the electron distribution function as a function of the initial energy γ_0 and initial position z_i at $t = 0$. The δ function localizes the integral on the dynamical trajectory $z(t,\gamma_0,z_i)$ with a transverse velocity $v_x(t,\gamma_0,z_i)$, given approximately by

$$v_x \simeq \frac{eA_W}{\gamma M c}, \tag{7}$$

where M is the electron mass, A_W is the wiggler vector potential, and where, for simplicity, we have assumed no transverse canonical momentum spread. We model the wiggler by

$$A_W = A_0 \sin(k_W z),$$

and, defining dimensionless vector potentials by $a = eA/Mc$, we find

$$\frac{da_m}{dt} \simeq -\frac{c l_m a_m}{2L_s} + \frac{\pi e^2 a_0}{M k_m L_s L_W}\int_0^T dt \int dz_i d\gamma_0 f_0(z_i,\gamma_0)\frac{\cos[k_m^+ z(t,z_i,\gamma_0) - k_m ct]}{\gamma(t,z_i,\gamma_0)}, \tag{9}$$

where we have defined $k_m^+ = k_m + k_W$. Up to this point the trajectories are unrestricted. We note, however, that this equation is a complicated nonlinear integral equation because of the dependence of the electron trajectories on the optical fields. To evaluate the integrals, we now expand the trajectories in the amplitude of the optical field and keep terms through third order. To compute the particle trajectories, it is convenient to use variables defined by $\Gamma \doteq \gamma^2$ and $w \doteq ct$. The trajectory equations then become

$$\frac{dz}{dw} = 1 - \frac{2 + a_0^2}{4\Gamma}, \tag{10}$$

$$\frac{d\Gamma}{dw} = -a_0 \sum_m k_m a_m \cos(k_m^+ z - k_m w). \tag{11}$$

Once the solutions of these equations are obtained as expansions, $z = z_0 + z_1 + z_2 + z_3 + \ldots$ and $\Gamma = \Gamma_0 + \Gamma_1 + \Gamma_2 + \Gamma_3 + \ldots$, in a_m, the dynamics portion of the integrand in eq. (9),

$$\mathfrak{J} = \frac{\cos[k_m^+ z(t, z_i, \gamma_0) - k_m ct]}{\gamma(t, z_i, \gamma_0)}, \tag{12}$$

is expanded to the same order, $\mathfrak{J} = \mathfrak{J}_0 + \mathfrak{J}_1 + \mathfrak{J}_2 + \mathfrak{J}_3 + \ldots$. Using the assumption of no optical prebunching and ignoring particle discreteness effects, the integrals of the zeroth order integrand $\mathfrak{J}_0$ and the second order integrand $\mathfrak{J}_2$ vanish; the first- and third-order terms are given by

$$\mathfrak{J}_1 = \frac{1}{\Gamma_0^{1/2}}\left[-\frac{\Gamma_1}{2\Gamma_0}\cos(k_m^+ z_0 - k_m w) - k_m^+ z_1 \sin(k_m^+ z_0 - k_m w)\right], \tag{13}$$

$$\begin{aligned} \mathfrak{J}_3 = \frac{1}{\Gamma_0^{1/2}}\Bigg[&\cos(k_m^+ z_0 - k_m w)\Big(-\frac{\Gamma_3}{2\Gamma_0} - \frac{5\Gamma_1^3}{16\Gamma_0^3} \\ &+ \frac{3\Gamma_1\Gamma_2}{4\Gamma_0^2} + \frac{\Gamma_1 k_m^{+2} z_1^2}{4\Gamma_0} - k_m^{+2} z_1 z_2\Big) + \\ &\sin(k_m^+ z_0 - k_m w)\Big(-k_m^+ z_3 + \frac{k_m^{+3} z_1^3}{6} \\ &+ \frac{\Gamma_1 k_m^+ z_2}{2\Gamma_0} + \frac{\Gamma_2 k_m^+ z_1}{2\Gamma_0} - \frac{\Gamma_1^2 k_m^+ z_1}{8\Gamma_0^2}\Big)\Bigg]. \end{aligned} \tag{14}$$

As a result of the vanishing of $\mathfrak{J}_0$ and $\mathfrak{J}_2$, one can infer that the spectral evolution equation has the form

$$\frac{da_m}{dt} = -\frac{cl_m}{2L_s} a_m + g_m a_m + a_m \sum_{n,r} \mathfrak{K}(m,n,r) a_n a_r, \tag{15}$$

where g_m is the small signal gain per pass and $\mathfrak{K}(m,n,r)$ is a kernel function that includes the effects of nonlinear saturation and sideband coupling.

3. Constant parameter wiggler

The expansions outlined in the previous section have been carried out in detail for the constant parameter wiggler used in the Los Alamos oscillator experiment. [1] In this case, the expansions of the position z and the squared energy Γ are most efficiently carried out by recognizing that eqs. (10) and (11) are derivable from a Hamiltonian

$$H = \Gamma - \frac{2 + a_0^2}{4} \ln \Gamma + \sum_m \epsilon_m \sin(k_m^+ z - k_m w), \tag{16}$$

with $\epsilon_m \equiv k_m a_m a_0 / k_m^+$. Using Hamilton-Jacobi perturbation theory to third order, the lowest order in the action S is given by

$$S_0 = \Gamma_0 z + c_2 w, \tag{17}$$

where

$$c_2 = \frac{2 + a_0^2}{4} \ln \Gamma_0 - \Gamma_0. \tag{18}$$

Defining supplementary variables $\bar{w} \equiv w$ and $\bar{z} \equiv z - \alpha w$ where $\alpha \equiv 1 - (2 + a_0^2)/(4\Gamma_0)$, the higher order terms in the action satisfy

$$\frac{\partial S_1}{\partial w} = -\sum_m \epsilon_m \sin[k_m^+ \bar{z} + (k_m^+ \alpha - k_m)\bar{w}], \tag{19}$$

$$\frac{\partial S_2}{\partial w} = \frac{2 + a_0^2}{8\Gamma_0^2} \left(\frac{\partial S_1}{\partial \bar{z}} \right)^2, \textit{and} \tag{20}$$

$$\frac{\partial S_3}{\partial w} = -\frac{2 + a_0^2}{12\Gamma_0^3} \left(\frac{\partial S_1}{\partial z} \right)^3 - \frac{2 + a_0^2}{4\Gamma_0^2} \frac{\partial S_1}{\partial \bar{z}} \frac{\partial S_2}{\partial z}. \tag{21}$$

Eq. (19) is easily integrated; given the solution, the right-hand side of eq. (20) is determined and can be integrated to give S_2. Given S_1 and S_2, the right-hand side of eq. (21) is determined and S_3 can then be determined. Given the action S through third order, the trajectories can then be determined through third order from

$$\Gamma = \frac{\partial S}{\partial z}. \tag{22}$$

$$z_i = \frac{\partial S}{\partial \Gamma_0}, \tag{23}$$

where (23) is an equation for the position z in terms of the initial position z_i and energy Γ. Given these solutions, the results are substituted in eqs. (13) and (14) to determine the required integrands. Although this procedure is straightforward, a very large number of terms result, especially at third order; therefore, all of the calculations were carried out using an algebraic manipulation computer code, SMP, a product of the Inference Corp. of Los Angeles, California. A number of consistency checks were applied to the calculation to ensure accuracy, including the identical vanishing of several thousand terms that would otherwise have been singular. Even though the results of these calculations were considerably shorter than the intermediate expressions, they were still long enough that their expression as FORTRAN code was also carried out in SMP to minimize the possibility of error.

The form of the evolution equation that results from these calculations is

$$\frac{da_m}{dt} = -\frac{cl_m}{2L_s}a_m + g_m a_m + a_m \sum_{n,r} K(m,n)a_n^2, \tag{24}$$

where $K(m,n)$ is a kernel function that includes both saturation and sideband effects. Defining the modal intensity by $i_m \equiv a_m^2$, eq. (24) can be rewritten as

$$\frac{di_m}{dt} = -\frac{cl_m}{L_s}i_m + g_m i_m + 2i_m \sum_{n,r} K(m,n)i_n. \tag{25}$$

Note that both the small signal gain term g_m and the nonlinear kernel $K(m,n)$ still contain information about the electron beam distribution function, including a linear proportionality to the current. Because of the optical loss term, $-cl_m i_m/L_s$, the dynamics in eq. (25) cannot be Hamiltonian. On the other hand, the form of the nonlinear term precludes the possibility that the dynamics is gradient-like with a simple evolution to steady state. Thus, we are led to expect that the dynamical behavior is generic, with steady state solutions, bifurcations to limit cles, and chaotic motions all being possible and relevant depending on the parameters

. That bifurcations to more complicated behavior are likely can be argued by observing that the gain and nonlinear terms in eq. (25) are both proportional to current. Thus. at sufficiently high current they dominate the system loss term. leading to the prediction that steady states become independent of current at sufficiently high current, i.e., putting more energy in the system gives no more optical power. Because this result is counterintuitive, one might expect that the steady state is irrelevant at very high current and that the spectrum is time ependen. even with zero detuning and steady electron beam current.

Although one can numerically integrate eq. (25) as an evolution equation starting from assumed initial conditions to determine the relevant nonlinear dynamics, we have not yet debugged the resulting code. Instead, we present results for the steady state, assuming zero detuning and the same electron beam distribution from one pass to the next; i.e., we investigate solutions of

$$\frac{cl_m}{2L_s} i_m = g_m i_m + i_m \sum_{n,r} K(m,n) i_n, \tag{26}$$

though recognizing the possible irrelevance of the solutions at very high current. Eq. (26) was solved numerically by starting from the assumption of diagonal dominance, i.e., ignoring the off-diagonal terms in $K(m,n)$ and iterating the equation including the off-diagonal terms using a Newton algorithm until the solution converged. The result for one case using the parameters of the oscillator experiment [1] with a peak current of 50 A is shown in fig. 1. A Gaussian distribution in energy with a full width of 1% and a parabolic distribution longitudinally were used in the integrals over the beam distribution function. The optical system loss function was assumed to be constant as a function of wavelength. Comparing fig. 1 with the experimental results, fig. 2 [1], for the smallest detuning it can be seen that the agreement is very good, including both the number of peaks in the spectrum and their location with respect to the peak of the small signal gain. Although it is difficult to extract the optical power from our one-dimensional model with any precision, a naive comparison using the quoted [1] value of the optical beam size gives the same power from the theory as was observed in the experiment, i.e., several hundred megawatts. The importance of nonlinear effects is obvious when it is observed that the peaks in the spectrum in fig. 1 do not lie on the peak of the small signal gain curve.

To test the possibility of sideband suppression using optical elements, a model system loss curve as shown in fig. 3 was used in a run with otherwise identical parameters; the results for the steady state spectrum are shown in fig. 4. As can be seen, the enhanced damping does result in a significant narrowing of the optical spectrum, but at the cost of a factor of approximately 9 in optical power as one would expect. Although this result is a very encouraging test for the theory, it should not be overinterpreted because of the obvious inadequacy of the model of optical system damping that was used.

4. Summary and discussion

We have presented an outline of the derivation of the spectral evolution equation for FEL oscillators and preliminary results on the steady state spectrum. The initial results compare very well with the experimental results both in the shape of the spectrum and in the magnitude of the steady state power.

There are a number of extensions and improvements of the theory that can obviously be done. The successful development of a code integrating eq. (25) would allow investigation of the start-up evolution and the effects of optical system detuning and varying beam current. Analytically and numerically, one can investigate the linear stability of the nonlinear saturated states of eq. (26) to determine when one might expect bifurcations to limit cycles or chaotic behavior. Finally, one can obviously extend the treatment of particle motion to include the effects of variable parameter wigglers; in this case one can anticipate that the resulting evolution equation will be considerably less useful than the results of ref. [2] for the design of wigglers, but might provide more insight (for a given moderate length wiggler) into the nature of the saturated spectrum.

Although the theory we have presented has a number of inherent limitations and requires a number of extensions or improvements, it has already demonstrated remarkably good agreement with experimental results and has shown that very significant sideband effects are predicted even for short wigglers with no clearly defined synchrotron oscillations.

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Figure captions

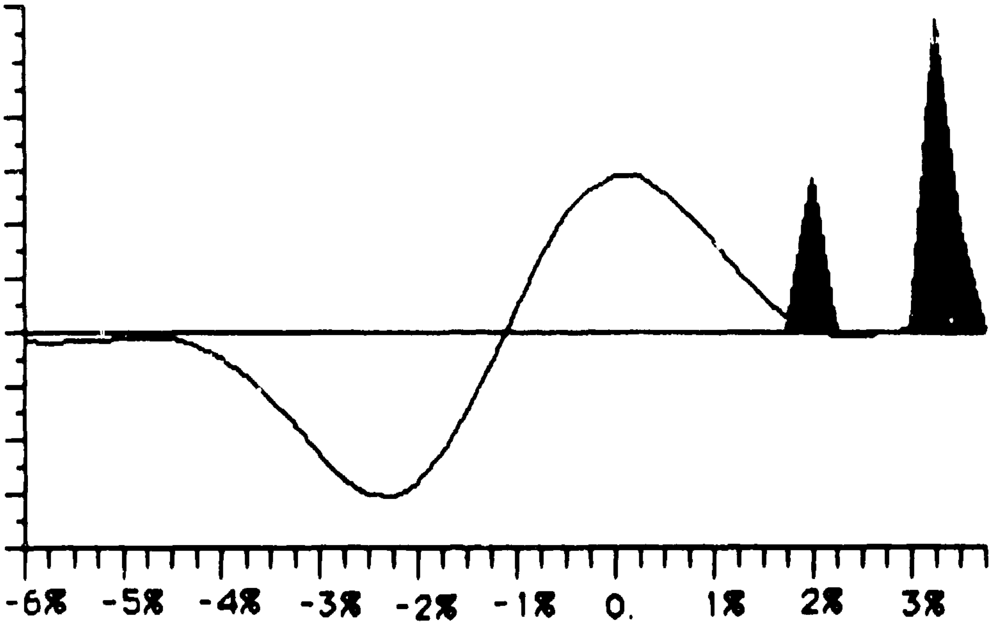
Fig. 1. Theoretical saturated spectrum and small signal gain as a function of fractional wavelength shift from the peak of the small signal gain.

Fig. 2. Experimental saturated spectrum as a function of fractional wavelength shift from the peak of the small signal gain.

Fig. 3. Model loss curve as function of wavelength shift used to test sideband suppression.

Fig. 4. Theoretical saturated spectrum and small signal gain as a function of

fractional wavelength shift from the peak of the small signal gain for model side-band suppression optics.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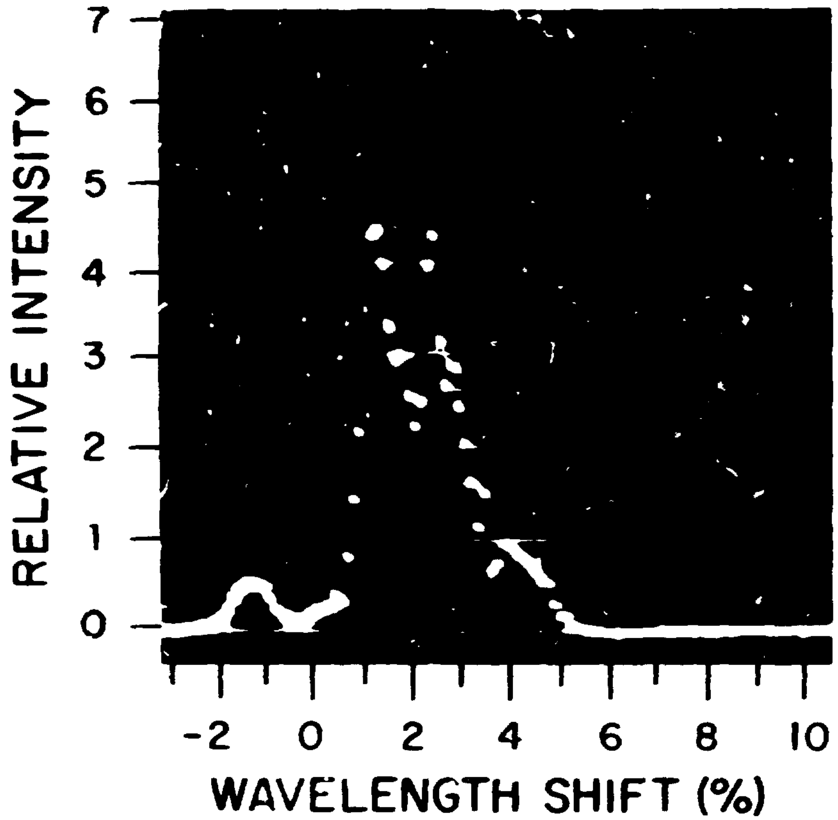

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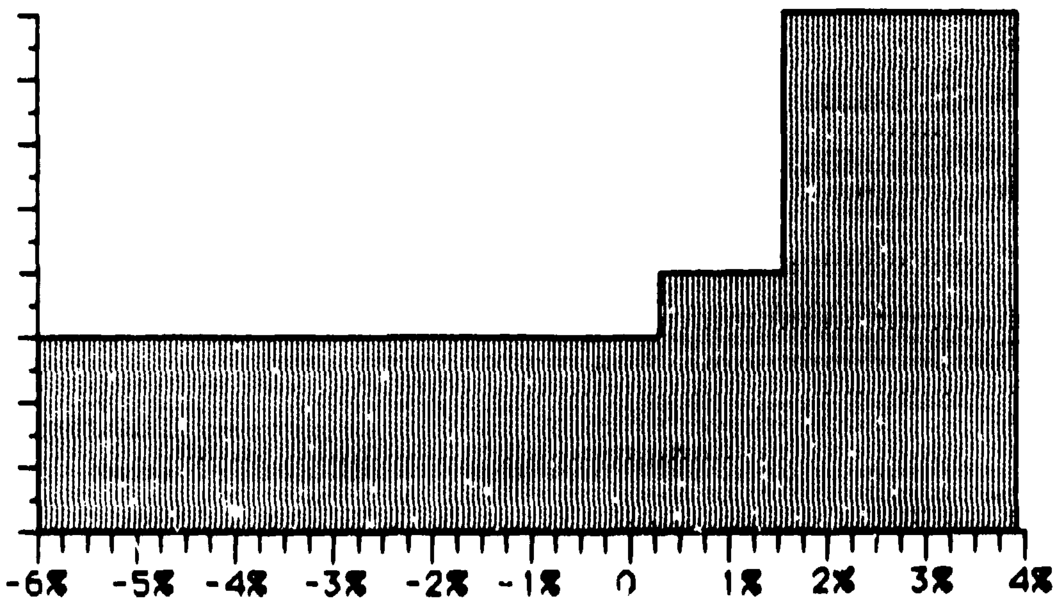

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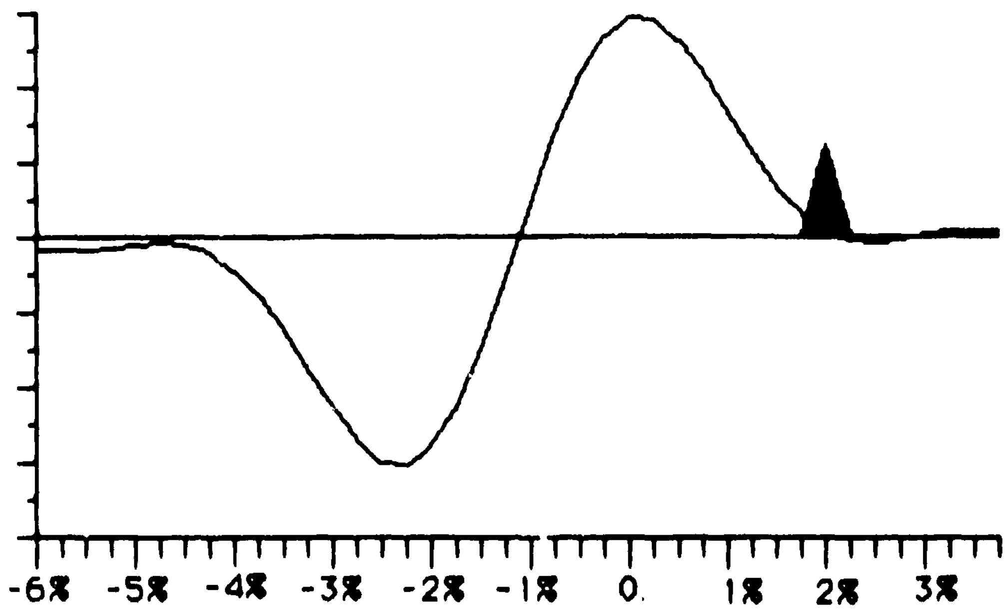

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